MODEL ANSWER ESE-2013-14
SUBJECT:- DECISION TECHNIQUE
CLASS:- B.TECH V SEM (I.P.E.)
CODE:- IPE-354

SECTION-A

A:-

1:- choose the correct Answer.

(i):- (c) Optimal utilization of existing resources

(ii):- (b) Outgoing variables

(iii):- (c) Square matrix

(iv):- (a) $u_i + v_j$

(v):- (c) Pure Game

(vi):- (a) All players act rationally and intelligently

(vii):- (a) Transient

(viii):- (c) Lottery draw

(ix) ~~(viii)~~:- (b) Sink Node

(x):- (a) ACTIVITY

B:-

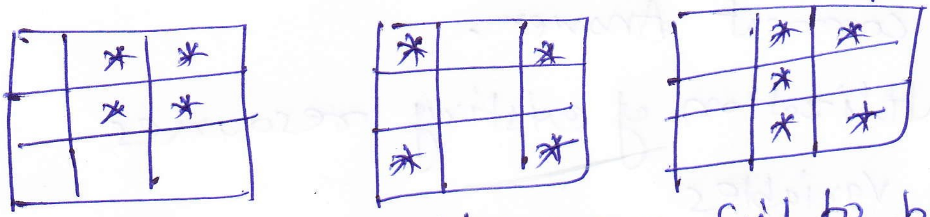
(a) Feasible and Non feasible solution of L.P.P:-

Ans:- Feasible solution:- A set of variables [x_1, x_2, --- x_{n+m}] is called a ~~feasible~~ feasible solution to L.P. problem if it is satisfies the constraints.

A set of variables is called a feasible solution to L.P. problem if it satisfies the constraints as well as non negativity restrictions.

(b) ~~Ditterence b/n~~ Degeneracy in T.P. :-

A basic feasible solution in which the total number of non negative allocations are less than m+n-1 is called degenerate basic feasible solution. and all the m+n-1 allocations are are in non- independent positions.



The allocations are said to be in independent positions, if it is impossible to form a closed path.

(c) Ditterence b/n PERT/CPM

CPM	PERT
① One time estimate	① Three time estimate
② Relastic in nature	② probabilistic in nature
③ Critical Path method	③ Program evaluation & Review technique
④ Less relastic & accurate	④ More accurate

(d) Queue discipline :-
ⓐ FIFO ⓑ LIFO ⓒ SIRO ⓓ Priority

~~(e) Do~~

Value of the game is

$$V = \frac{a|c-d| + c|a-b|}{|a-b| + |c-d|} = \frac{6\times4 + 8\times3}{4+3}$$

$$V = \frac{48}{7}$$

Hence, the strategies of player A is $= \left(\frac{4}{7} \; \& \; \frac{3}{7}\right)$

———— " ———— player B is $= \left(\frac{5}{7}, \frac{2}{7}\right)$

Value of the game $= \frac{48}{7} = 6\frac{6}{7}$

Dominance property :-

In some games, it is possible to reduce the size of the payoff matrix by eliminating redundant rows or columns.

If a game has such redundant rows or columns, those row or columns are dominated by some other rows or columns, respectively such property is known as dominance property

Dominance property of rows:-

(a) In the payoff matrix of player A, if all the entries in row X are greater than or equal to the corresponding entries of another row Y then row Y is dominated by row X. Under such condition, row Y of the payoff matrix can be deleted.

(b) In payoff matrix of player A, if each of the sum of entries of any two rows (sum of the entries of row X and row Y) is greater than or equal to corresponding entry of a third row Z, then row Z is dominated by row X and row Y. Under this situation row Z can be deleted.

Dominance property of columns:-

(a) In the payoff matrix of player A, if all the entries in a column α are lesser than or equal to corresponding entries of another column y, then y is dominated by column α. Under this condition column y can be deleted.

(b) In the payoff matrix of player A, if each of the sum of the entries of any two columns (sum of α and y) is lesser than or equal to the corresponding entry of third column Z, then column Z is dominated by α & y.

ANSWER:-02

method.

Solution. Let x_1 and x_2 be the grams of food X and Y to be purchased. Then the problem can be formulated as follows :

$$\text{Minimize } Z = 12x_1 + 20x_2,$$
$$\text{subject to} \quad 6x_1 + 8x_2 \geq 100,$$
$$7x_1 + 12x_2 \geq 120,$$
$$x_1, x_2 \geq 0.$$

Step 1. Express the problem in standard form

Slack variables s_1 and s_2 are subtracted from the left-hand sides of the constraints to convert them to equations. These variables are also called *negative slack variables* or *surplus variables*. Variable s_1 represents units of vitamin A in product mix *in excess* of the minimum requirement of 100, s_2 represents units of vitamin B in product mix *in excess* of requirement of 120. Since they represent 'free' foods, the cost coefficients associated with them in the objective function are zeros. The problem, therefore, can be written as follows :

$$\text{Minimize } Z = 12x_1 + 20x_2 + 0s_1 + 0s_2,$$
$$\text{subject to} \quad 6x_1 + 8x_2 - s_1 = 100,$$
$$7x_1 + 12x_2 - s_2 = 120,$$
$$x_1, x_2, s_1, s_2 \geq 0.$$

Step 2. Find initial basic feasible solution

Putting $x_1 = x_2 = 0$, we get $s_1 = -100$, $s_2 = -120$ as the first basic solution but it is not feasible as s_1 and s_2 have negative values that do not satisfy the non-negativity restrictions. Therefore, we introduce artificial variables A_1 and A_2 in the constraints, which take the form

$$6x_1 + 8x_2 - s_1 + A_1 = 100,$$
$$7x_1 + 12x_2 - s_2 + A_2 = 120,$$
$$x_1, x_2, s_1, s_2, A_1, A_2 \geq 0.$$

Now artificial variables with values greater than zero violate the equality in constraints established in step 1. Therefore, A_1 and A_2 should not appear in the final solution. To achieve this, they are assigned a large unit penalty (a large positive value, + M) in the objective function, which can be written as

$$\text{minimize } Z = 12x_1 + 20x_2 + 0s_1 + 0s_2 + MA_1 + MA_2.$$

Problem, now, has six variables and two constraints. Four of the variables have to be zeroised to get initial basic feasible solution to the 'artificial system'. Putting $x_1 = x_2 = s_1 = s_2 = 0$, we get

$$A_1 = 100, A_2 = 120, Z = 220\text{ M}.$$

Note that we are starting with a very heavy cost (compare it with zero profit in maximization problem) which we shall minimize during the solution procedure. Table 2.36 represents the problem and its solution.

Step 3: Perform optimality test

TABLE 2.36

c_B	c_j / Basis	12 x_1	20 x_2	0 s_1	0 s_2	M A_1	M A_2	b	θ
M	A_1	6	8	−1	0	1	0	100	25/2
M	A_2	7	(12)	0	−1	0	1	120	10 ←
	Z_j	13M	20M	−M	−M	M	M	220 M	
	c_j-Z_j	12-13M	20-20M	M	M	0	0		
			↑K						

Initial solution

Since c_j-Z_j is *negative* under x_1, x_2-columns, initial solution is not optimal and can be improved. c_j-Z_j is most negative under x_2-column. x_2-column is the key column, A_2-row is the key row and (12) is the key element. Since A_2 is leaving variable, column A_2 is deleted from the next table.

Step 4: Iterate towards on optimal solution

Performing iterations results in the following tables:

TABLE 2.37

c_B	c_j / Basis	12 x_1	20 x_2	0 s_1	0 s_2	M A_1	b	θ
M	A_1	$(4/3)$	0	−1	$2/3$	1	20	15 ←
20	x_2	$7/12$	1	0	$-1/12$	0	10	$120/7$
	Z_j	$\frac{35}{3}+\frac{4}{3}M$	20	−M	$-\frac{5}{3}+\frac{2}{3}M$	M	200 + 20M	
	c_j-Z_j	$\frac{1}{3}-\frac{4}{3}M$	0	M	$\frac{5}{3}-\frac{2}{3}M$	0		
		↑K						

c_B	Basis	x_1	x_2	s_1	s_2	b
12	x_1	1	0	$-3/4$	$1/2$	15
20	x_2	0	1	$7/16$	$-3/8$	$5/4$
	Z_j	12	20	$-1/4$	$-3/2$	205
	c_j-Z_j	0	0	$1/4$	$3/2$	

∴ Optimal solution is

$x_1 = 15$, $x_2 = 5/4$; $Z_{min} = 205$ Paise = Rs. 2.05.

Hence 15 grams of food X and $\frac{5}{4}$ grams of food Y should be the required product mix with minimum cost of Rs. 2.05.

EXAMPLE 2.17-2

Maximize $Z = 3x_1 - x_2$,

subject to $2x_1 + x_2 \le 2$,

ANSWER NO:-03

Maximize $Z = x_1 + 2x_2 + 3x_3 - x_4$,

subject to
$$x_1 + 2x_2 + 3x_3 = 15,$$
$$2x_1 + x_2 + 5x_3 = 20,$$
$$x_1 + 2x_2 + x_3 + x_4 = 10,$$
$$x_1, x_2, x_3, x_4 \geq 0.$$

[*M.D.U.B.E. (Mech.) Dec., 2006; P.T.U.B.E. (Mech.) 2010; May, 2006, C.Sc. 2009; P.U.B.B.A, 2001*]

Solution

Step 1. Introducing artificial variables A_1, A_2, A_3, the given problem in standard form is

maximize $Z = x_1 + 2x_2 + 3x_3 - x_4 - MA_1 - MA_2 - MA_3$,

subject to
$$x_1 + 2x_2 + 3x_3 + 0x_4 + A_1 + 0A_2 + 0A_3 = 15,$$
$$2x_1 + x_2 + 5x_3 + 0x_4 + 0A_1 + A_2 + 0A_3 = 20,$$
$$x_1 + 2x_2 + x_3 + x_4 + 0A_1 + 0A_2 + A_3 = 10,$$
$$x_1, x_2, x_3, x_4, A_1, A_2, A_3 \geq 0.$$

Step 2. Initial basic (non-degenerate) solution to the artificial system is

$x_1 = x_2 = x_3 = x_4 = 0,$
$$A_1 = 15,$$
$$A_2 = 20,$$
$$A_3 = 10,$$
$$Z = -45\text{ M}.$$

Table 2.41 represents this solution.

TABLE 2.41

	c_j	1	2	3	−1	−M	−M	−M		
c_B	Basis	x_1	x_2	x_3	x_4	A_1	A_2	A_3	b	θ
−M	A_1	1	2	3	0	1	0	0	15	5
−M	A_2	2	1	(5)	0	0	1	0	20	4 ←
−M	A_3	1	2	1	1	0	0	1	10	10
	Z_j	−4M	−5M	−9M	−M	−M	−M	−M	−45M	
	$c_j - Z_j$	1 + 4M	2 + 5M	3 + 9M	−1 + M	0	0	0		
				↑						

Initial solution

Since $c_j - Z_j$ is positive under some variable columns, table 2.41 is not optimal.

Step 3. Performing iterations to get an optimal solution results in the following tables

TABLE 2.42

	c_j	1	2	3	−1	−M	−M		
c_B	Basis	x_1	x_2	x_3	x_4	A_1	A_3	b	θ
−M	A_1	$-1/5$	$(7/5)$	0	0	1	0	3	$15/7$ ←
3	x_3	$2/5$	$1/5$	1	0	0	0	4	20
−M	A_3	$3/5$	$9/5$	0	1	0	1	6	$10/3$
	Z_j	$\frac{6-2M}{5}$	$\frac{3-16M}{5}$	3	−M	−M	−M	12−9M	
	$c_j - Z_j$	$\frac{-1+2M}{5}$	$\frac{7+16M}{5}$ ↑	0	−1 + M	0	0		

Second solution

TABLE 2.43

	c_j	1	2	3	−1	−M		
c_B	Basis	x_1	x_2	x_3	x_4	A_3	b	θ
2	x_2	$-1/7$	1	0	0	0	$15/7$	∞
3	x_3	$3/7$	0	1	0	0	$25/7$	∞
−M	A_3	$6/7$	0	0	(1)	1	$15/7$	$15/7$ ←
$Z_j = \Sigma c_B a_{ij}$		$\frac{7-6M}{7}$	2	3	−M	−M	$\frac{105-15M}{7}$	
	$c_j - Z_j$		0	0	−1 + M ↑	0		

Third solution

TABLE 2.44

	c_j	1	2	3	−1		
c_B	Basis	x_1	x_2	x_3	x_4	b	θ
2	x_2	$-1/7$	1	0	0	$15/7$	−15
3	x_3	$3/7$	0	1	0	$25/7$	$25/3$
−1	x_4	$(6/7)$	0	0	1	$15/7$	$5/2$ ←
$Z_j = \Sigma c_B a_{ij}$		$1/7$	2	3	−1	$90/7$	
	$c_j - Z_j$	$6/7$ ↑	0	0	0		

4th basic feasible solution

TABLE 2.45

	c_j	1	2	3	−1	
c_B	Basis	x_1	x_2	x_3	x_4	b
2	x_2	0	1	0	$1/6$	$5/2$

3	x_3	0	0	1	$-\frac{1}{2}$	$\frac{5}{2}$
1	x_1	1	0	0	$\frac{7}{6}$	$\frac{5}{2}$
$Z_j = \Sigma c_B a_{ij}$		1	2	3	0	15
	$c_j - Z_j$	0	0	0	-1	

Optimal basic feasible solution

Since $c_j - Z_j$ is either zero or negative under all columns, the optimal basic feasible solution has been attained. Optimal values are

$$x_1 = \frac{5}{2}, x_2 = \frac{5}{2}, x_3 = \frac{5}{2}, x_4 = 0.$$

$$A_1 = A_2 = A_3 = 0 \text{ and } Z_{max} = 15.$$

EXAMPLE [illegible]

The Air Force is experimenting with three types of bombs P, Q and R in which three kinds of explosives, viz. A, B and C will be used. Taking the various factors into account, it has been decided to use the maximum 600 kg of explosive A, at least 480 kg of explosive B and exactly 540 kg of explosive C. Bomb P requires 3, 2, 2 kg, bomb Q requires 1, 4, 3 kg and bomb R requires 4, 2, 3 kg of explosives A, B and C respectively. Bomb P is estimated to give the equivalent of a 2 ton explosion, bomb Q, a 3 ton explosion and bomb R, a 4 ton explosion respectively. Under what production schedule can the Air Force make the biggest bang ?

[SVSM PGDM, 2009; P.U.B. Tech. (T.I.T.) Dec., 2008]

Solution. Let x_1, x_2 and x_3 be the number of bombs of type P, Q and R respectively. Then the linear programming model for the problem is given by

$$\text{maximize } Z = 2x_1 + 3x_2 + 4x_3,$$
$$\text{subject to} \quad 3x_1 + x_2 + 4x_3 \le 600,$$
$$2x_1 + 4x_2 + 2x_3 \ge 480,$$
$$2x_1 + 3x_2 + 3x_3 = 540,$$
$$x_1, x_2, x_3 \ge 0.$$

Step 1. Introducing slack, surplus and artificial variables, the problem is expressed in the following standard form :

$$\text{Maximize } Z = 2x_1 + 3x_2 + 4x_3 + 0s_1 + 0s_2 - MA_1 - MA_2,$$
$$\text{subject to} \quad 3x_1 + x_2 + 4x_3 + s_1 = 600,$$
$$2x_1 + 4x_2 + 2x_3 - s_2 + A_1 = 480,$$
$$2x_1 + 3x_2 + 3x_3 + A_2 = 540,$$
$$x_1, x_2, x_3, s_1, s_2, A_1, A_2 \ge 0.$$

Step 2. Substituting $x_1 = x_2 = s_2 = 0$ in the above artificial system, the following initial solution is obtained :

$$s_1 = 600, A_1 = 480, A_2 = 540 ; Z = -1{,}020 \text{ M}.$$

Step 3. Table 2.46 represents the above information. This table is not optimal and can be improved.

Step 4. Successive iterations are represented in the subsequent tables.

TABLE 2.46

c_B	c_j	2	3	4	0	0	− M	− M		
	Basis	x_1	x_2	x_3	s_1	s_2	A_1	A_2	b	θ
0	s_1	3	1	4	1	0	0	0	600	600
− M	A_1	2	(4)	2	0	− 1	1	0	480	120←
− M	A_2	2	3	3	0	0	0	1	540	180
	Z_j	− 4M	− 7M	− 5M	0	M	− M	− M	−1,020M	
	$c_j - Z_j$	2 + 4M	3 + 7M	4 + 5M	0	− M	0	0		
			↑							

Initial solution

Table 3.160

To / *From*	*D*	*E*	*F*	*G*
A	6	7	8	10
B	4	10	7	6
C	3	22	2	11

Determine the optimum distribution for the company to minimize costs.

[*P.U.B.E.(Mech.) 1997*]

Solution

Step 1: Set up the Transportation Table

Overtime production can be represented as additional factories, producing the item at their corresponding higher costs. For istance, shipping cost for the overtime shipment from factory A to warehouses D, E, F and G will be 6 + 5 = 11, 7 + 5 = 12, 8 + 5 = 13 and 10 + 5 = 15 rupees respectively. Similarly, from factory B to these warehouses it will be Rs. 8, 14, 11 and 10 respectively and from C to them it will be Rs. 9, 28, 8 and 17 respectively. Hypothetical factories for overtime production will have capacities of 70, 80 and 10 units respectively. As the total capacity is higher than the total demand, a dummy warehouse *d* is created to absorb this excess capacity [illegible] + 190) – (165 + 175 + 205 + 165) = 140 units. The modified matrix is shown in table 3.161.

Table 3.161

To / *From*	D	E	F	G	*d*	*Capacity*
A	6	7 (175)	8 (25)	10	0 (30)	230
B	4 (115)	10	7	6 (165)	0	280
C	3	22	2 (180)	11	0	180
A_1	11	12	13	15	0 (70)	70
B_1	8 (50)	14	11	10	0 (30)	80
C_1	9	28	8	17	0 (10)	10
Demand	165	175	205	165	140	*Initial b.f.s*

Step 2: Find the Initial Basic Feasible Solution

This solution obtained by following the Vogel's approximation method is shown in table 3.161.

Step 3: Perform Optimality Test

The above solution is not optimal. The optimal solution after one iteration is shown in table 3.162.

Table 3.162

To / From	D	E	F	G	d	Capacity
A	6 (30)	7 (175)	8 (25)	10	0	230
B	4 (115)	10	7	6 (165)	0	280
C	3	22	2 (180)	11	0	180
A_1	11	12	13	15	0 (70)	70
B_1	8 (20)	14	11	10	0 (60)	80
C_1	9	28	8	17	0 (10)	10
Demand	165	175	205	165	140	*Optimal solution*

3.7 ADDITIONAL PROBLEMS

EXAMPLE 3.7-1

A company has seven manufacturing units situated in different parts of the country ... recession it is proposed to close four of these and to concentrate production in the ... three units. Production in these units will actually be increased from present levels ... require an increase in the personnel employed in them. Personnel at the closed units ... their desire for moving to any one of the remaining units and the company is willing ... them removal (transfer) costs.

The retraining expenses would have to be incurred as the technology in these ... different. Not all existing personnel can be absorbed by transfer and a number of ... will arise, cost of redundancy is given as a general figure at each unit closed.

		A	B	C
No. employed :		200	400	300

(These units A, B, C and D are ...)

Retraining costs in Rs. '000/person Transfer to		A	B	C
	E	0.5	0.4	0.6
	F	0.6	0.4	0.6
	G	0.5	0.3	0.7

Removal costs in Rs. '000/person Transfer to		A	B	C
	E	2.5	3.6	3.4
	F	2.4	4.6	3.4
	G	2.5	2.7	3.3

		A	B	C
Redundancy payments in Rs. '000/person	:	6.0	5.0	6.0

		E	F	G
Additional personnel required at units Required	:	350	450	200

(i) Obtain a solution to the problem of the cheapest means to transfer personnel ... units closed to those units which will be expanded.

ANSWER NO:-05

(15)

Player A and B play a game in which each player has three coins (20p, 25p and 50p). Each of them select a coin without the knowledge of the other person. If the sum of the values of the coin is an even number, A win's B's coin. If that sum is an odd number, B wins A's coin.

(a) Develop a payoff matrix with respect to player A.

(b) Find the optimal strategies for the players.

Solution:- Payoff matrix:-

		Player B q_1 (20p)	q_2 (25p)	q_3 (50p)	Row minimum
Player A	P_1 (20p)	20	-20	50	-20
	P_2 (25p)	-25	25	-25	-25
	P_3 (50p)	20	-50	50	-50
	Column maximum	20	25	50	

There is no saddle point.

Check for the dominance property:-

Row III is dominated by Row-I, hence Row III is to be deleted.

New matrix after deleting Row - II

Player A vs Player B

	q_1 20p	q_2 25p	q_3 50p
P_1 20p	20	−20	50
P_2 25p	−25	25	−25

then again column III is dominated by column - I then column III is to be deleted

Payoff matrix after deleting column III

Player B

Player A	q_1 (20p)	q_2 (25p)	oddments	
P_1 20p	20	−20	50	$\lvert c-d \rvert$
P_2 25p	−25	25	40	$\lvert a-b \rvert$
oddments	45	40		
	$\lvert b-d \rvert$	$\lvert a-c \rvert$		

$$P_1 = \frac{50}{50+40} = \frac{5}{9}, \quad P_2 = \frac{40}{90} = \frac{4}{9}$$

$$q_1 = \frac{45}{45+45} = \frac{1}{2} \qquad q_2 = \frac{45}{45+45} = \frac{1}{2}$$

value of the game is $V = \dfrac{20\times 50 - 25\times 40}{50+40} = 0$

Hence, the strategies of player A & B & the value of game

$$A = \left(\frac{5}{9}, \frac{4}{9}, 0\right), \quad B = \left(\frac{1}{2}, \frac{1}{2}, 0\right), \quad V = 0$$

ANSWER NO:-06

(13)

Consider the 4x4 game played by players A & B and solve it optimally.

Player B

Player A	1	2	3	4
1	6	2	4	8
2	2	-1	1	12
3	2	3	3	9
4	5	2	6	10

Solution:- Payoff matrix with maximin & minimax value

B

A	1	2	3	4	Row minima
1	6	2	4	8	2 (maximin)
2	2	-1	1	12	-1
3	2	3	3	9	2 (maximin)
4	5	2	6	10	2 (maximin)
Column maxima	6	3 (minimax)	6	12	

Game has no saddle point :-

Check for dominance property :-

for row there is no dominance.

for column :- Sum of the values in column I and column II is lesser than or equal to corresponding value in the column IV. Hence column I & II dominate column 4. So column IV is to be deleted and the corresponding result is :-

Payoff matrix after deleting column IV

Player A (rows) / B (columns)

	1	2	3
1	6	2	4
2	2	-1	1
3	2	3	3
4	5	2	6

In this payoff sum of value ~~of~~ in row I & row III is greater than or equal to the corresponding value in row II, Hence Row II is to be deleted.

Payoff matrix after deleting row II

Player A (rows) / B (columns)

	1	2	3
1	6	2	4
2	2	3	3
4	5	2	6

Again:- Values in column II are less than or equal to the corresponding values in column III. Hence column II dominate column III and column III can be deleted

Payoff matrix after deleting column III

		Player B 1	2
Player A	1	6	2
	3	2	3
	4	5	2

Now, values in row I are greater than or equal to row IV, row I dominates row IV so row IV can be deleted.

Payoff matrix after deleting Row IV

		Player B 1	2	oddments
Player A	1	6	2	1
	3	2	3	4
	oddments	1	4	

$p_1 = \frac{1}{1+4} = 1/5$

$p_2 = \frac{4}{5}$

$q_1 = \frac{1}{5}$

$q_2 = \frac{4}{5}$

$$V = \frac{6*1 + 2*4}{1+4} = \frac{14}{5} = \underline{\underline{2.8}}$$

The optimal solution of the problem is

$$A\left(\frac{1}{5}, 0, \frac{4}{5}, 0\right), \quad B\left(\frac{1}{5}, \frac{4}{5}, 0, 0\right)$$

value = $\underline{\underline{2.8}}$

ANSWER NO:-08

...-service store employs one cashier at its counter. Nine customers arrive on an average ...minutes while the cashier can serve 10 customers in 5 minutes. Assuming Poisson ... for arrival rate and exponential distribution for service time, find

... average number of customers in the system.
... average number of customers in the queue or average queue length.
... average time a customer spends in the system.
... average time a customer waits before being served.

[P.T.U. B.E., 2001; Karn. U. B.E. (Mech.) 1998, 95]

Solution

Arrival rate $\lambda = 9/5 = 1.8$ customers/minute,
service rate $\mu = 10/5 = 2$ customers/minute.

1. Average number of customers in the system,

$$L_s = \frac{\lambda}{\mu - \lambda} = \frac{1.8}{2 - 1.8} = 9.$$

2. Average number of customers in the queue,

$$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)} = \frac{\lambda}{\mu} \cdot \frac{\lambda}{(\mu - \lambda)} = \frac{1.8}{2} \times \frac{1.8}{2 - 1.8} = 8.1.$$

3. Average time a customer spends in the system,

$$W_s = \frac{1}{\mu - \lambda} = \frac{1}{2 - 1.8} = 5 \text{ minutes.}$$

4. Average time a costumer waits in the queue,

$$W_q = \frac{\lambda}{\mu}\left(\frac{1}{\mu - \lambda}\right) = \frac{1.8}{2}\left(\frac{1}{2 - 1.8}\right) = 4.5 \text{ minutes}$$

EXAMPLE 10.9-4.2

A person repairing radios finds that the time spent on the radio sets has exponential distribution with mean 20 minutes. If the radios are repaired in the order in which they come in and their arrival is approximately Poisson with an average rate of 15 for 8-hour day, what is the repairman's expected idle time each day ? How many jobs are ahead of the average set just brought in ?

[P.U.B.E. (T&I.T.) Nov., 2004; B.E. (Mech.) 2002 ; P.T.U. B. (Tech.) 2010; 2000; MBA May, 2002 ; IGNOU MBA 2000; G.J.U. B.E. (Mech.) 1996]

Solution

$$\text{Arrival rate } \lambda = \frac{15}{8 \times 60} = \frac{1}{32} \text{ units/minute,}$$

ANSWER NO:-09

Customers arrive at the First Class Ticket Counter of a Theatre at the rate of ... There is one clerk serving the customers at the rate of 30 per hour.

(i) What is the probability that there is no customer in the counter (i.e. that the ... idle) ?

(ii) What is the probability that there are more than 2 customers in the counter ?

(iii) What is the probability that there is no customer waiting to be served ?

(iv) What is the probability that a customer is being served and no body is waiting ?

[SVSM PGDM, 2009; P.U.B.E. (T.I.T.) Dec., 2008; P.T.U. ...

Solution

Here, $\lambda = 12\text{/hour}, \mu = 30\text{/hour}.$

(i) Probability that there is no customer in the system,

$$p_0 = 1 - \frac{\lambda}{\mu} = 1 - \frac{12}{30} = 0.6.$$

(ii) Probability that there are more than two customers in the counter

$$= p_3 + p_4 + p_5 + \ldots$$

$$= 1 - (p_0 + p_1 + p_2)$$

$$= 1 - \left[p_0 \left(1 + \frac{\lambda}{\mu} + \frac{\lambda^2}{\mu^2}\right) \right]$$

$$= 1 - \left[0.6 \left(1 + \frac{12}{30} + \frac{144}{900}\right) \right] = 0.064.$$

(iii) Probability that there is no customer waiting to be served

= probability that there is at most one customer in the system

$$= p_0 + p_1 = 0.6 + 0.6 \left(\frac{12}{30}\right) = 0.84.$$

(iv) Probability that a customer is being served and no body is waiting $= p_1$ ...

$$= 0.6 \left[\frac{12}{30}\right] = 0.24.$$

EXAMPLE 10.9-4.13

In the central railway station 15 computerised reservation counters are available ... customer can book his/her ticket in any train on any day in any one of these computerised reservation counters. The average time spent per customer by each clerk is 5 minutes ... arrivals per hour during three types of activity periods have been calculated and customers have been surveyed to determine how long they are willing to wait during each type of period.

Type of period	*Arrivals/hr*	*Customer's acceptable waiting time*
Peak	*110*	*15 minutes*
Normal	*60*	*10 minutes*
Low	*30*	*5 minutes*

Making suitable assumptions on this queuing process, determine how many counters should be kept open during each type of period.

[M.D.U. Rohtak B.E. (Mech.) Dec. ...

ANSWER NO:-10

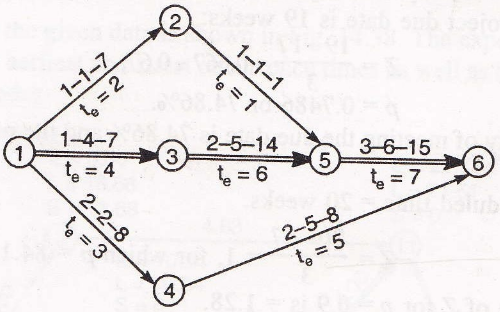


Fig. 14.36

(b) For determining the expected project length, the expected activity times need to be calculated. The same, along with the variances, are computed below.

Activity	t_0	t_m	t_p	$t_e = \dfrac{t_o + 4t_m + t_p}{6}$	$\sigma^2 = \left(\dfrac{t_p - t_o}{6}\right)^2$
1-2	1	1	7	2	1
1-3	1	4	7	4	1
1-4	2	2	8	3	1
2-5	1	1	1	1	0
3-5	2	5	14	6	4
4-6	2	5	8	5	1
5-6	3	6	15	7	4

Length of path 1-2-5-6 = 2 + 1 + 7 = 10,
length of path 1-3-5-6 = 4 + 6 + 7 = 17, and
length of path 1-4-6 = 3 + 5 = 8.

Since 1-3-5-6 has the longest duration, it is the critical path of the network.

∴ The expected project length = 17 weeks.

(c) Variance of the project length is the sum of the variances of the activities on the critical

$$V_{cp} = V_{1\text{-}3} + V_{3\text{-}5} + V_{5\text{-}6} = 1+4+4 = 9.$$

$$\therefore \quad \sigma = 3 \text{ weeks.}$$

(*d*) (*i*) Probability that the project will be completed at least 4 weeks earlier than expected time:

$$\text{Expected time} = 17 \text{ weeks,}$$

and $$\text{scheduled time} = 17-4 = 13 \text{ weeks.}$$

∴ The standard normal deviate,

$$Z = \frac{13-17}{3} = -1.33.$$

For Z = –1.33, probability is 1 – 0.9082 = 0.0918 or the probability of completing the project at least 4 weeks earlier than expected time *i.e.*, within 13 weeks.

(*ii*) Probability that the project will be completed no more than 4 weeks later than expected time:

$$\text{Expected time} = 17 \text{ weeks.}$$

$$\therefore \quad \text{Scheduled time} = 17 + 4 = 21 \text{ weeks.}$$

$$\therefore \quad Z = \frac{21-17}{3} = 1.33.$$

$$\therefore \quad p = 0.9082.$$

Therefore, the probability of completing the project in not more that 21 weeks is 90.82%.

(*e*) When the project due date is 19 weeks:

$$Z = \frac{19-17}{3} = 0.667 \approx 0.67,$$

for which, $$p = 0.7486 \text{ or } 74.86\%.$$

∴ The probability of meeting the due date is 74.86% and the probability of not meeting the due date is 25.14%.

(*f*) Scheduled time = 20 weeks.

$$\therefore \quad Z = \frac{20-17}{3} = 1, \text{ for which } p = 84.13\%.$$

(*g*) Value of *Z* for *p* = 0.9 is = 1.28.

$$\therefore \quad 1.28 = \frac{T-17}{3} \text{ or } T = 17 + 3.84 = 20.84 \text{ weeks.}$$

EXAMPLE 14.13-6

A PERT network is shown in Fig. 14.37. The activity times in days are given along the arrows. The scheduled times for some important events are given along the nodes. Determine the critical path and probabilities of meeting the scheduled dates for the specified events. Tabulate the results and determine slack for each event.

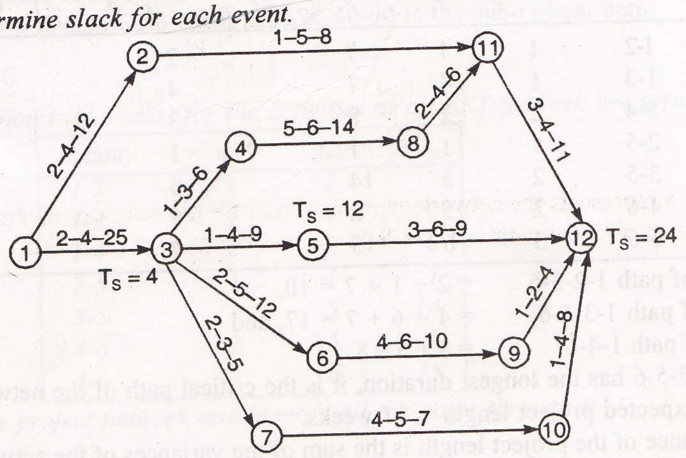


Fig. 14.37

[*H.P.U. B. Tech. (Mech.) June, 2007*]

ANSWER NO:-11

...cost slope for each activity and the normal direct cost of the project is calculated. This is shown in the table below.

Activity	:	1-2	1-3	2-4	2-5	3-4	4-6	5-6	6-7
Cost slope (Rs. /day)	:	20	70	50	10	55	55	30	35

(*i*) Next, the network is drawn and the critical path is determined. This is shown in Fig.

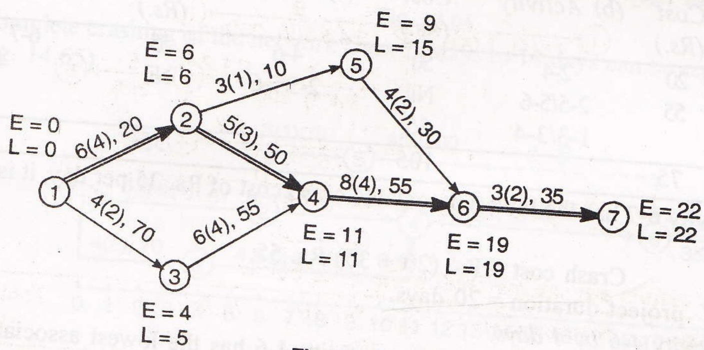


Fig. 14.59

(*ii*) The critical path is 1-2-4-6-7.

(*iii*) Normal duration = 22 days.

Normal cost = Rs. (470 + 22 × 10) = Rs. 690.

Now represent the network on time-scaled diagram. This is shown in Fig. 14.60.

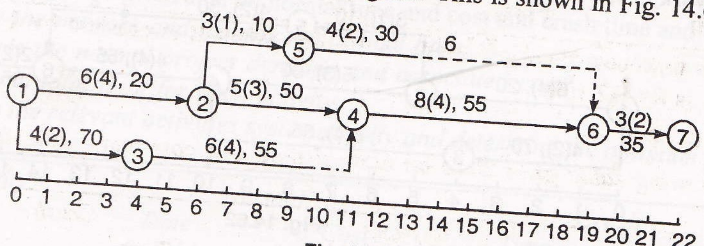


Fig. 14.60

Crash activity 1-2 by 1 day.

The various alternative activities and their crash costs are given below.

(a) Activity	Cost (Rs.)	(b) Activity	Cost (Rs.)	(c) Activity	Cost (Rs.)	(d) Activity	Cost (Rs.)
1-2	20	2-4	50	4-6	55	6-7	35
1-3/3-4	Nil	2-5/5-6	Nil	2-5/5-6	Nil		
		1-3/3-4	Nil				
	20		50		55		35

Since activity 1-2 has the lowest associated crash cost of Rs. 20 per day, it is crashed by one day.

Crash cost = Rs. 20,

project duration = 21 days.

The network is shown in Fig. 14.61

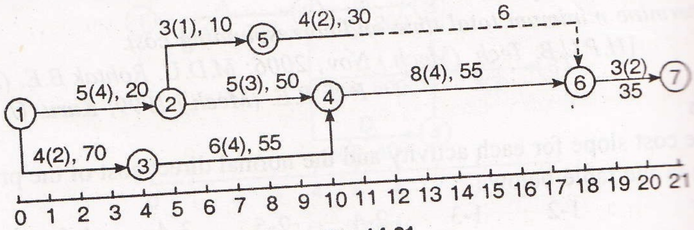


Fig. 14.61

Crash activity 6-7 by 1 day.

The various alternative activities and their crash costs are given below.

(a) Activity	Cost (Rs.)	(b) Activity	Cost (Rs.)	(c) Activity	Cost (Rs.)	(d) Activity	Cost (Rs.)
1-2	20	2-4	50	4-6	55	6-7	35
1-3/3-4	55	2-5/5-6	Nil	2-5/5-6	Nil		
		1-3/3-4	55				
	75		105		55		35

Since activity 6-7 has the lowest associated crash cost of Rs. 35 per day, it is crashed by one day.

Crash cost = Rs. (20 + 35) Rs. 55,

∴ project duration = 20 days.

Crash activity 4-6 by 4 days.

As evident from the above table, next activity 4-6 has the lowest associated crash cost of Rs. 55 per day and as seen from Fig. 14.61 it can be crashed by 4 days.

Crash cost = Rs. (55 + 4 × 55) = Rs. 275,

∴ project duration = 16 days.

The network is shown in Fig. 14.62.

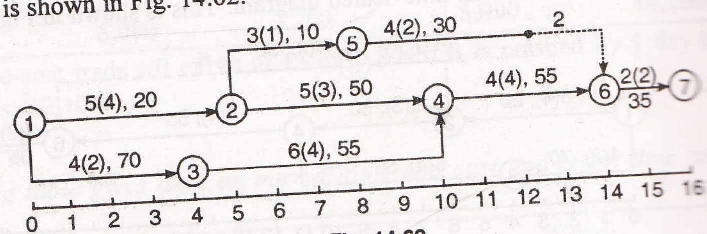


Fig. 14.62

Crash activity 1-2 by 1 day.

Next, activity 1-2 is crashed by 1 day at a cost of Rs. 75 per day.

Crash cost = Rs. (275 + 75) = Rs. 350,

∴ project duration = 15 days.

The network is shown in Fig. 14.63.

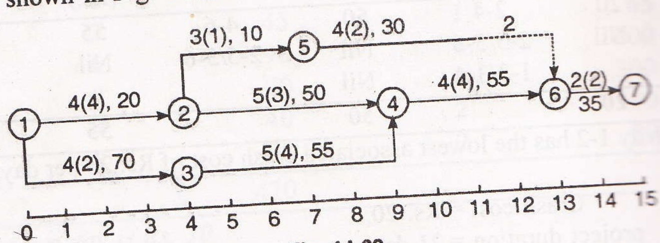


Fig. 14.63

Crash activity 2-4 by 1 day.

Now activity 2-4 is crashed by 1 day at a cost of Rs. 105 per day.

∴ Crash cost = Rs. (350 + 105) = Rs. 455,

project duration = 14 days.

The network is shown in Fig. 14.64. No further crashing is possible.

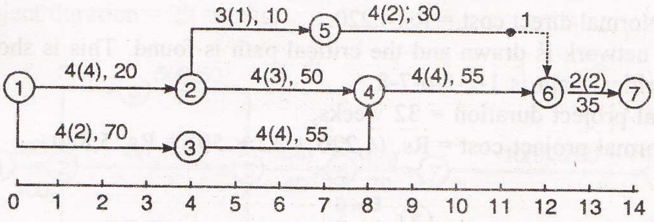


Fig. 14.64

The complete crashing of the network from 22 days to 14 days can be represented in a single diagram (Fig. 14.65).

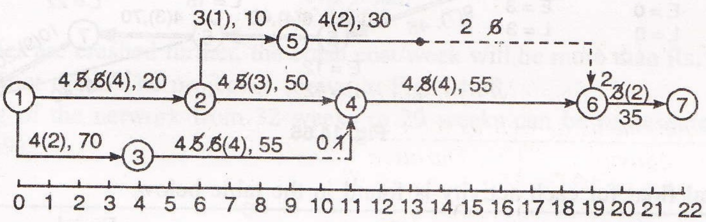


Fig. 14.65

∴ Minimum total time = 14 days,

corresponding cost = Rs. (690 + 455) = Rs. 1,145.

EXAMPLE 14.14-3

The following table gives data on normal time and cost and crash time and cost for a project.

(a) Draw the network and identify the critical path.

(b) What is the normal project duration and associated cost?

(c) Find out total float for each activity.

(d) Crash the relevant activities systematically and determine the optimum project time and cost.

Activity	*Normal*		*Crash*	
	Time (weeks)	*Cost (Rs.)*	*Time (weeks)*	*Cost (Rs.)*
1-2	*3*	*300*	*2*	*400*
2-3	*3*	*30*	*3*	*30*
2-4	*7*	*420*	*5*	*580*
2-5	*9*	*720*	*7*	*810*
3-5	*5*	*250*	*4*	*300*
4-5	*0*	*0*	*0*	*0*
5-6	*6*	*320*	*4*	*410*
5-7	*4*	*400*	*3*	*470*
6-8	*13*	*780*	*10*	*900*
7-8	*10*	*1,000*	*9*	*1,200*
		4,220		

Indirect costs are Rs. 50 per week. [*I.C.W.A. (Final) Dec., 1988*]